IMAC Special Session on Algebraic Structures in the Numerical Analysis of Differential Equations

IMAC, Universitat Jaume I, Castellón, Spain

31 May 2012

Schedule

09:30 – 09:45	Presentation
09:45 – 10:45	**Kurusch Ebrahimi-Fard**, ICMAT, Madrid. *The Magnus expansion, trees and Knuth's rotation correspondence*
10:45 – 11:15	Coffee break
11:15 – 12:15	**Gilles Vilmart**, E.N.S. Cachan, Antenne de Bretagne, France. *Modified differential equations for stochastic differential equations*
12:15 – 13:15	**Reinout Quispel***, La Trobe University, Australia. *T.b.a.*
15:15 – 16:15	**Ander Murua**, Euskal Herriko Unibertsitatea, San Sebastián *Formal series for highly oscillatory systems*

*To be confirmed.

Abstracts

The Magnus expansion, trees and Knuth's rotation
Kurusch Ebrahimi-Fard

In numerical analysis the successful use of combinatorics on trees can be traced back to the pioneering work of John Butcher on an algebraic theory of integration methods. Since then, exploring and unfolding algebraic structures in the context of the theory of numerical integration methods became a useful tool. In this talk we report on recent progress made in the understanding of the fine structure of the so-called Magnus expansion. The latter is a peculiar Lie series involving Bernoulli numbers, iterated Lie brackets and integrals. It results from the recursive solution of a particular differential equation, which was introduced by Wilhelm Magnus in 1954, and which characterizes the logarithm of the solution of linear initial value problems for linear operators. Arieh Iserles and collaborators were the first to use planar tree structures in an intriguing way to study the Magnus expansion. Our work is based on using simple combinatorics on planar rooted trees, which allows us to prove a closed formula for the Magnus expansion in the context of free dendriform algebra. From this, by using a well-known dendriform algebra structure on the vector space generated by the disjoint union of the symmetric groups, we derive the Mielnik–Plebanski–Strichartz formula for the continuous Baker–Campbell–Hausdorff series.

Modified differential equations for stochastic differential equations
Gilles Vilmart

Inspired by recent advances in the theory of modified equations, we present a new methodology for constructing numerical integrators with high weak order for the time integration of stochastic differential equations (SDEs).

This approach is illustrated with the constructions of new methods of weak order two with good qualitative properties: implicit integrators that exactly conserve all quadratic first integrals, and implicit and stabilized explicit integrators suitable for stiff SDEs (S-ROCK type methods).

The framework of trees and related algebraic structures was not used to derive these results. However, the Taylor series involved can become very complicated especially for high orders of the methods and for high dimensions of the problem. I would like to discuss the opportunity of using such algebraic tools in this context.

This is a joint work with Assyr Abdulle, David Cohen, and Konstantinos C. Zygalakis.

Formal series for highly oscillatory systems

Ander Murua

We show that B-series and related formal series expansions that are nowadays used to analyze numerical integrators provide a powerful means to study and implement the method of averaging. We consider systems with $d \geq 1$ constant fast frequencies of the form

$$\frac{d}{dt}y = \epsilon f(y, t\omega), \quad y(0) = y_0 \in \mathbb{R}^D,$$

where $f(y, \theta)$ depends 2π-periodically on each of the scalar components $\theta_1, \ldots, \theta_d$ of the angular variable $\theta \in \mathbb{T}^d$ and $\omega \in \mathbb{R}^d$ is a constant vector of angular frequencies. We assume that ω is *non-resonant*, i.e. $k \cdot \omega \neq 0$ for each multi-index $k \in \mathbb{Z}^d$, with $k \neq \mathbf{0}$.

Our approach makes it possible to construct in an *explicit* way higher-order averaged systems and the associated changes of variables. In our new approach, the averaged system and the change of variables consist of vector-valued functions that may be written down immediately and scalar *coefficients* that are universal in the sense that they do not depend on the specific system being averaged and may therefore be computed once and for all for each vector of frequencies $\omega \in \mathbb{R}^d$.